Modeling Technology Product Revenue
Technology Companies - Part I

Gary Schurman MBE, CFA

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Products in the high-tech space evolve to accommodate competitive market pressures, rapid rates of technology change, and constant improvements in performance and functionality. While adding functionality and value, the fast moving technologies also make products obsolete quickly. Technological obsolescence usually occurs when a new product or technology supersedes the old, and it becomes preferred to use the new technology in place of the old, even if the old product is still functional. Typically, obsolescence is preceded by a gradual decline in popularity. For example, the average life cycle of a typical semiconductor device is approximately three years, which includes introduction, production, and end-of-life phases.

In this white paper we will model product revenue over time on a single product that is subject to technological obsolescense. To that end we will use the following hypothetical problem...

Our Hypothetical Problem

Assume that we are tasked with estimating product net operating income for a product introduced into the market at the end of month 6 (time s) of the current year. We are given the following model parameters...

Table 1: Model Parameters

Symbol	Description	Value
R_s^s	Annualized product revenue time s (\$)	10,000,000
θ	Pre-tax revenue margin (%)	35.00
α	Income tax rate (%)	12.50
β	Weighted average revenue life in years (#)	3.00

Question: What is product after-tax net operating income in years one through five?

Annualized Revenue

Products in the high-tech space evolve to accommodate competitive market pressures, rapid rates of technology change, and constant improvements in performance and functionality. While adding functionality and value, the fast moving technologies also make products obsolete quickly. Technological obsolescence usually occurs when a new product or technology supersedes the old, and it becomes preferred to use the new technology in place of the old, even if the old product is still functional. For example, the average life cycle of a typical semiconductor device is approximately three years, which includes introduction, production, and end-of-life phases.

We will define the variable R_t^s to be annualized revenue at time t on a product brought to market at time s, and the variable λ to be the rate of technological obsolescense. The equation for annualized revenue at time t is...

$$R_t^s = R_s^s \,\mathrm{Exp}\left\{ -\lambda\,(t-s) \right\} \text{ ...where... } s \leq t \tag{1}$$

We will define the variable $R^s_{m,n}$ to be cumulative revenue realized over the time interval $[m, n]$ on a product brought to market at time s. Using Equation (1) above, the equation for cumulative revenue is...

$$R^s_{m,n} = \int\limits_m^n R^s_t \,\delta t = R^s_s \int\limits_m^n \mathrm{Exp}\left\{ -\lambda\,(t-s)\right\} \delta t \quad \text{...where...} \quad s \le m \le n \tag{2}$$

Using Appendix Equation (18) below, the solution to Equation (2) above is...

$$R^s_{m,n} = R^s_s \left(\mathrm{Exp}\left\{ -\lambda\,(m-s)\right\} - \mathrm{Exp}\left\{ -\lambda\,(n-s)\right\}\right) \lambda^{-1} \tag{3}$$

After-Tax Revenue Margin

We will define the variable θ to be the pre-tax revenue margin. The definition of pre-tax revenue margin is...

$$\theta = \frac{\text{Operating revenue - Operating expense}}{\text{Operating revenue}} \tag{4}$$

We will define the variable $N^s_{m,n}$ to be after-tax net operating income realized over the time interval $[m, n]$ on a product brought to market at time s, and the variable α to be the income tax rate. Using Equation (1) above, the equation for cumulative net income is...

$$N^s_{m,n} = \int\limits_m^n \theta\,(1-\alpha)\,R^s_t \,\delta t = \theta\,(1-\alpha)\,R^s_s \int\limits_m^n \mathrm{Exp}\left\{ -\lambda\,(t-s)\right\} \delta t \tag{5}$$

Using Equation (3) above, the solution to Equation (5) above is...

$$N^s_{m,n} = \theta\,(1-\alpha)\,R^s_s \left(\mathrm{Exp}\left\{ -\lambda\,(m-s)\right\} - \mathrm{Exp}\left\{ -\lambda\,(n-s)\right\}\right) \lambda^{-1} \tag{6}$$

Note that if we are given GAAP after-tax revenue margin for any given period then using Equation (6) above, the equation for annualized revenue at the beginning of the period is...

$$R^0_0 = \lambda\,N^0_{0,\Delta} \left[\theta\,(1-\alpha) \left(1 - \mathrm{Exp}\left\{ -\lambda\,\Delta \right\}\right)\right]^{-1} \quad \text{...where...} \quad \Delta = \text{Period length in years} \tag{7}$$

Note that GAAP after-tax revenue margin in Equation (7) above is defined as GAAP net income excluding research and development expense, amortization of intangibles expense, and non-operating items (example: interest income and expense).

Weighted Average Revenue Life

We will define the variable β to be weighted average revenue life in years. We will define weighted average revenue life to be time-weighted revenue realized over the time interval $[s, \infty]$ divided by unweighted revenue realized over that same time interval. The equation for weighted average revenue life is...

$$\beta = \int\limits_s^\infty (t-s)\,R^s_t \,\delta t \Bigg/ \int\limits_s^\infty R^s_t \,\delta t \tag{8}$$

Using Equation (1) above, we can rewrite the numerator of Equation (8) above as...

$$\begin{aligned} \int\limits_s^\infty (t-s)\,R^s_t \,\delta t &= \int\limits_s^\infty (t-s)\,R^s_s\,\mathrm{Exp}\left\{ -\lambda\,(t-s)\right\} \delta t \\ &= \int\limits_s^\infty (t-s)\,R^s_s\,\mathrm{Exp}\left\{ -\lambda\,t\right\} \mathrm{Exp}\left\{ \lambda\,s\right\} \delta t \\ &= R^s_s\,\mathrm{Exp}\left\{ \lambda\,s\right\} \int\limits_s^\infty (t-s)\,\mathrm{Exp}\left\{ -\lambda\,t\right\} \delta t \end{aligned} \tag{9}$$

Using Appendix Equations (25) below, the solution to Equation (9) above is...

$$R_s^s \,\mathrm{Exp}\left\{\lambda\, s\right\}\mathrm{Exp}\left\{-\lambda\, s\right\}\lambda^{-2} = R_s^s\,\lambda^{-2} \tag{10}$$

Using Equation (1) above, we can rewrite the numerator of Equation (8) above is...

$$\int\limits_s^\infty R_t^s\,\delta t = R_s^s \int\limits_s^\infty \mathrm{Exp}\left\{-\lambda\,(t-s)\right\}\delta t = R_s^s\,\mathrm{Exp}\left\{\lambda\, s\right\}\int\limits_s^\infty \mathrm{Exp}\left\{-\lambda\, t\right\}\delta t \tag{11}$$

Using Appendix Equations (18) below, the solution to Equation (11) above is...

$$\int\limits_s^\infty R_t^s\,\delta t = R_s^s\,\mathrm{Exp}\left\{\lambda\, s\right\}\mathrm{Exp}\left\{-\lambda\, s\right\}\lambda^{-1} = R_s^s\,\lambda^{-1} \tag{12}$$

Using Equations (10) and (12) above, the solution to Equation (8) above is...

$$\beta = \frac{R_s^s\,\lambda^{-2}}{R_s^s\,\lambda^{-1}} = \lambda^{-1} \tag{13}$$

Using Equation (13) above, the equation for the rate of technological obsolescense is...

$$\text{if...}\;\; \beta = \lambda^{-1}\;\;\text{...then...}\;\; \lambda = \frac{1}{\beta} \tag{14}$$

The Answer To Our Hypothetical Problem

Question: What is product after-tax net operating income in years one through five?

Using Equation (14) above and the data in Table 1 above, the equation for the rate of technological obsolescense is...

$$\lambda = \frac{1}{3.00} = 0.3333 \tag{15}$$

Using Equations (6) and (15) above and the data in Table 1 above, the answer to the question is...

Year	Net Income
1	1,410,000
2	2,205,000
3	1,580,000
4	1,132,000
5	811,000

Example: Year one: $s = \frac{6}{12} = 0.50, m = \frac{6}{12} = 0.50, n = \frac{12}{12} = 1.00$

$$N_{m,n}^s = 0.3500 \times (1 - 0.1250) \times 10,000,000 \times \Bigg(\mathrm{Exp}\left\{-0.3333 \times (0.50 - 0.50)\right\} - \mathrm{Exp}\left\{-0.3333 \times (1.00 - 0.50)\right\}\Bigg) \times 0.3333^{-1} = 1,410,000 \tag{16}$$

Example: Year three: $s = \frac{6}{12} = 0.50, m = 2.00, n = 3.00$

$$N_2^3 = 0.3500 \times (1 - 0.1250) \times 10,000,000 \times \Bigg(\mathrm{Exp}\left\{-0.3333 \times (2.00 - 0.50)\right\} - \mathrm{Exp}\left\{-0.3333 \times (3.00 - 0.50)\right\}\Bigg) \times 0.3333^{-1} = 1,580,000 \tag{17}$$

Appendix

A. The solution to the following integral is...

$$\int_{s}^{t} \mathrm{Exp}\left\{-\lambda\, u\right\} \delta u = -\frac{1}{\lambda}\, \mathrm{Exp}\left\{-\lambda\, u\right\} \Big[_{s}^{t}$$
$$= -\frac{1}{\lambda}\left(\mathrm{Exp}\left\{-\lambda\, t\right\} - \mathrm{Exp}\left\{-\lambda\, s\right\}\right)$$
$$= \frac{1}{\lambda}\left(\mathrm{Exp}\left\{-\lambda\, s\right\} - \mathrm{Exp}\left\{-\lambda\, t\right\}\right) \tag{18}$$

Note that when the upper bound of the integral above is infinity the solution to that integral becomes...

$$\int_{s}^{\infty} \mathrm{Exp}\left\{-\lambda\, t\right\} \delta t = \frac{1}{\lambda}\left(\mathrm{Exp}\left\{-\lambda\, s\right\} - \mathrm{Exp}\left\{-\lambda\, \infty\right\}\right) = \frac{1}{\lambda}\, \mathrm{Exp}\left\{-\lambda\, s\right\} \tag{19}$$

B. The solution to the following integral is...

$$I = \int_{s}^{\infty} (t-s)\, \mathrm{Exp}\left\{-\lambda\, t\right\} \delta t \tag{20}$$

We will define the function $f(t)$ as follows...

$$f(t) = t - s \;\text{ ...such that... }\; \frac{\delta f(t)}{\delta t} = 1 \tag{21}$$

We will define the function $g(t)$ as follows...

$$g(t) = -\frac{1}{\lambda}\, \mathrm{Exp}\left\{-\lambda\, t\right\} \;\text{ ...where... }\; \frac{\delta g(t)}{\delta t} = \mathrm{Exp}\left\{-\lambda\, t\right\} \tag{22}$$

Using Appendix Equations (21) and (22) above we can rewrite Appendix Equation (20) above as...

$$I = \int_{s}^{\infty} f(t)\, \frac{\delta g(t)}{\delta t}\, \delta t \tag{23}$$

Note that Equation (23) above cannot be solved as written. Using Equations (21) and (22) above we will rewrite that equation via integration by parts as...

$$I = f(t)\, g(t) \Big[_{s}^{\infty} - \int_{s}^{\infty} g(t)\, \frac{\delta f(t)}{\delta t}\, \delta t = -\frac{1}{\lambda}\, (t-s)\, \mathrm{Exp}\left\{-\lambda\, t\right\} \Big[_{s}^{\infty} + \frac{1}{\lambda} \int_{s}^{\infty} \mathrm{Exp}\left\{-\lambda\, t\right\} \delta t \tag{24}$$

Using Equation (18) above (noting that the first half of the equation is zero when evaluated at t and ∞) we can rewrite Equation (24) above as...

$$I = \frac{1}{\lambda^2}\left(\mathrm{Exp}\left\{-\lambda\, s\right\} - \mathrm{Exp}\left\{-\lambda\, \infty\right\}\right) = \frac{1}{\lambda^2}\, \mathrm{Exp}\left\{-\lambda\, s\right\} \tag{25}$$